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Moments of polarized parton distribution functions

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Abstract. We present novel results for the first moment of the spin-dependent structure function $g_1(x, Q^2)$ of the nucleon at "small" ($Q^2 < 0.3$ GeV^2) photon virtuality in the framework of a relativistic formulation of baryon chiral perturbation theory. We perform a next-to-leading order calculation and obtain significant differences to previously found results based on the heavy-baryon approach for the proton and neutron.

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1 Introduction

The spin structure of the nucleon is under active theoretical and experimental investigation. Of particular interest is the transition from the perturbative regime at high momentum transfer to the non-perturbative low-energy region. Interestingly, systematic and controlled theoretical calculations are only available for very small or very large momentum transfer, while the intermediate region is accessible to resonance models or can be investigated using dispersion relations. Of special interest is the generalized Gerasimov-Drell-Hearn (GDH) sum rule [1] for virtual photons. It was pointed out in [2] that one can systematically investigate its momentum evolution at low energies in the framework of chiral perturbation theory (ChPT). In a series of papers, Ji *et al.* [3,4] proposed a formalism that is better suited for the direct connection with the spin-dependent structure functions measured in deep inelastic scattering. A leading-order calculation was presented in [4] for the first moment of the spin-dependent nucleon structure function $g_1(x, Q^2)$ and it was found that the Q^2 variation was much stronger than expected from naive dimensional analysis. Here we present the results of a recent next-to-leading order calculation [5] of the first moment $\Gamma_1(Q^2)$ for proton and neutron targets based on a relativistic formulation of ChPT [6].

2 The first moment of $\mathbf{g_1(x, Q^2)}$

We start from the spin amplitude of forward doubly virtual Compton scattering (VVCS), which can be written in terms of two structure functions, called $S_1(\nu, Q^2)$ and $S_2(\nu, Q^2)$:

$$T^{[\mu\nu]}(p,q,s) = -i\,\epsilon^{\mu\nu\alpha\beta} q_\alpha \Big\{ m s_\beta S_1(\nu,Q^2) + [p\cdot q\, s_\beta - s\cdot q\, p_\beta]\,\frac{S_2(\nu,Q^2)}{m} \Big\}, \qquad (1)$$

where s^μ denotes the spin-polarization four-vector of the nucleon, m is the nucleon mass, $\epsilon^{\mu\nu\alpha\beta}$ the totally antisymmetric Levi-Civita tensor with $\epsilon^{0123} = 1$, $Q^2 \geq 0$ (the negative of) the photon virtuality and $\nu = p\cdot q/m$ the energy transfer. In what follows, we will be concerned with the amplitudes $\bar{S}_i(0, Q^2) = S_i(0, Q^2) - S_i^{\mathrm{el}}(0, Q^2)$, *i.e.* the Compton amplitudes with the contribution from the elastic intermediate state subtracted. Here we are interested in the so-called "first-moment" sum rule $\Gamma_1(Q^2)$, which via the optical theorem is connected with the VVCS structure function $\bar{S}_1(\nu, Q^2)$ by

$$\frac{2}{Q^2}\int_0^{x_0} \mathrm{d}x\, g_1(x,Q^2) = \frac{1}{4e^2}\,\bar{S}_1(0,Q^2) = \frac{2}{Q^2}\,\Gamma_1(Q^2)\ , \quad (2)$$

with $g_1(x = Q^2/2m\nu, Q^2)$ denoting the spin-dependent structure function of the nucleon. Furthermore, x_0 denotes the inelastic threshold. The subtracted Compton amplitude $\bar{S}_1(\nu, Q^2)$ is normalized as $\bar{S}_1(0, 0) = -e^2\kappa^2/m^2$, with κ the anomalous magnetic moment of the spin-1/2 particle. Equation (2) therefore constitutes one of the possible generalizations of the GDH sum rule to non-zero Q^2. At small photon virtualities, the subtracted Compton amplitudes $\bar{S}_i(0, Q^2)$ ($i = 1, 2$) can be calculated in ChPT and thus allow us to give predictions for $\Gamma_1(Q^2)$ via eq. (2).

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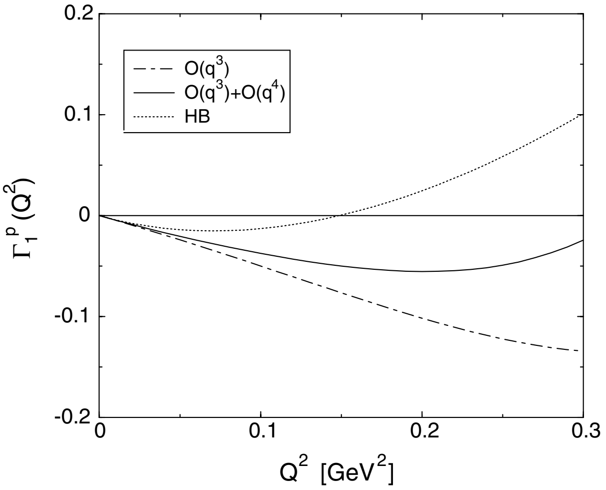

Fig. 1. Four-momentum dependence of the first moment $\Gamma_1(Q^2)$ for a proton target. The solid (dot-dashed) line gives the result of the present calculation to order q^3 (q^4) in comparison to the heavy-baryon result of [4] (dotted line).

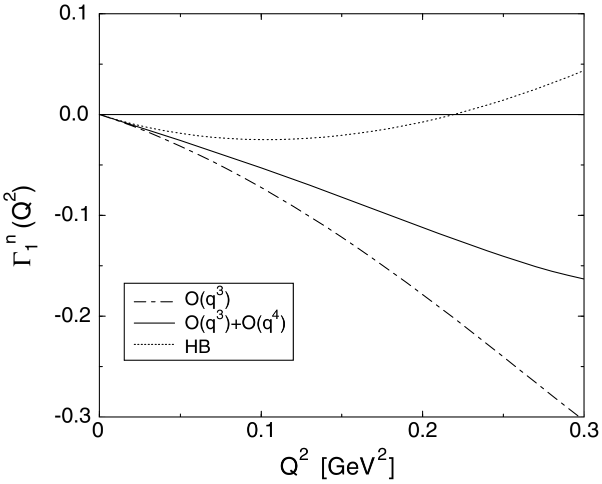

Fig. 2. Four-momentum dependence of the first moment $\Gamma_1(Q^2)$ for a neutron target. The plot symbols are explained in fig. 1.

3 Results

In fig. 1 we show $\Gamma_1(Q^2)$ for a proton target. We note that in the heavy-baryon framework of ref. [4] the $\mathcal{O}(p^4)$ calculation shown as the dotted line constitutes the leading-order (LO) result for the four-momentum dependence of $\Gamma_1(Q^2)$, whereas in the relativistic framework [5] the leading-order contribution to $\Gamma_1(Q^2)$ already starts at $\mathcal{O}(p^3)$. We note that for $Q^2 < 0.2$ GeV^2 the next-to-leading order (NLO) result of the relativistic calculation represents a small correction to the leading-order one. However, comparing the NLO relativistic with the LO heavy-baryon result, one notices large differences (*e.g.*, the delayed zero crossing in the relativistic calculation). Given that the heavy-baryon results can be obtained from the relativistic calculation by taking the nucleon mass to infinity, one must conclude that the $1/m$ expansion of heavy-baryon ChPT converges very slowly for this spin-dependent observable.

In fig. 2 we show $\Gamma_1(Q^2)$ for a neutron target. Once more both the LO and NLO relativistic results compared to the LO heavy-baryon result are compared. As in the case of the proton, the relativistic chiral expansion seems under good control, whereas the heavy-baryon result is remarkably different, showing even a zero crossing in the low-Q^2 domain ! We note that as in the case of the proton all curves have the same slope near $Q^2 = 0$, as the (real-photon) GDH sum-rule is satisfied (by construction) in all these calculations.

Finally, in fig. 3 we show $\Gamma_1(Q^2)$ for the isovector difference proton-neutron. At very large Q^2 this moment is related to the well-known Bjorken sum rule. In the low-Q^2 resonance region this quantity is also of interest because it is free from $\Delta(1232)$ pole contributions. It has therefore been suggested [7] that the isovector channel at low Q^2 can be calculated quite reliably in ChPT, without the

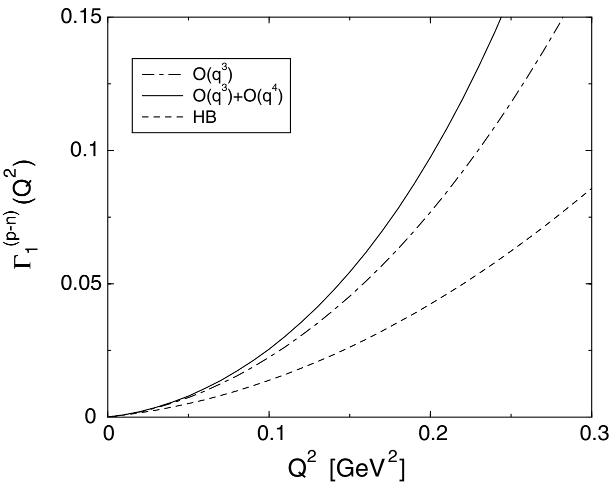

Fig. 3. Four-momentum dependence of the first moment $\Gamma_1(Q^2)$ for the isovector combination proton-neutron. The plot symbols are explained in fig. 1.

need of employing explicit delta degrees of freedom[1]. However, again comparing the NLO relativistic and the LO heavy baryon result, one must conclude that the isovector channel is not only plagued by a slowly converging $1/m$ expansion, but that additional physics beyond the one-loop pion-nucleon continuum seems to play an important role, as the NLO relativistic calculation rises far too quickly to smoothly meet the Bjorken limit at large Q^2. Clearly these issues need to be further investigated, necessitating investigations with (relativistic) Chiral Effective Field Theories that go beyond pions and nucleons as the only explicit degrees of freedom [8].

[1] We note that all the $\Gamma_1(Q^2)$ ChPT calculations for proton and neutron shown in figs. 1, 2 only contain the one-loop pion-nucleon continuum contribution. Possible contributions from nucleon or vector meson resonance are not shown.

References

1. S.B. Gerasimov, Yad. Fiz. **2**, 598 (1965); S.D. Drell, A.C. Hearn, Phys. Rev. Lett. **16**, 908 (1966).
2. V. Bernard, N. Kaiser, Ulf-G. Meißner, Phys. Rev. D **48**, 3062 (1993).
3. X.D. Ji, J. Osborne, J. Phys. G **27**, 127 (2001).
4. X.D. Ji, C.W. Kao, J. Osborne, Phys. Lett. B **472**, 1 (2000).
5. V. Bernard, T.R. Hemmert, U.-G. Meißner, Phys. Lett. B **545**, 105 (2002).
6. T. Becher, H. Leutwyler, Eur. Phys. J. C **9**, 643 (1999).
7. V.D. Burkert, Phys. Rev. D **63**, 097904 (2001).
8. V. Bernard, T.R. Hemmert, U.-G. Meißner, in preparation.